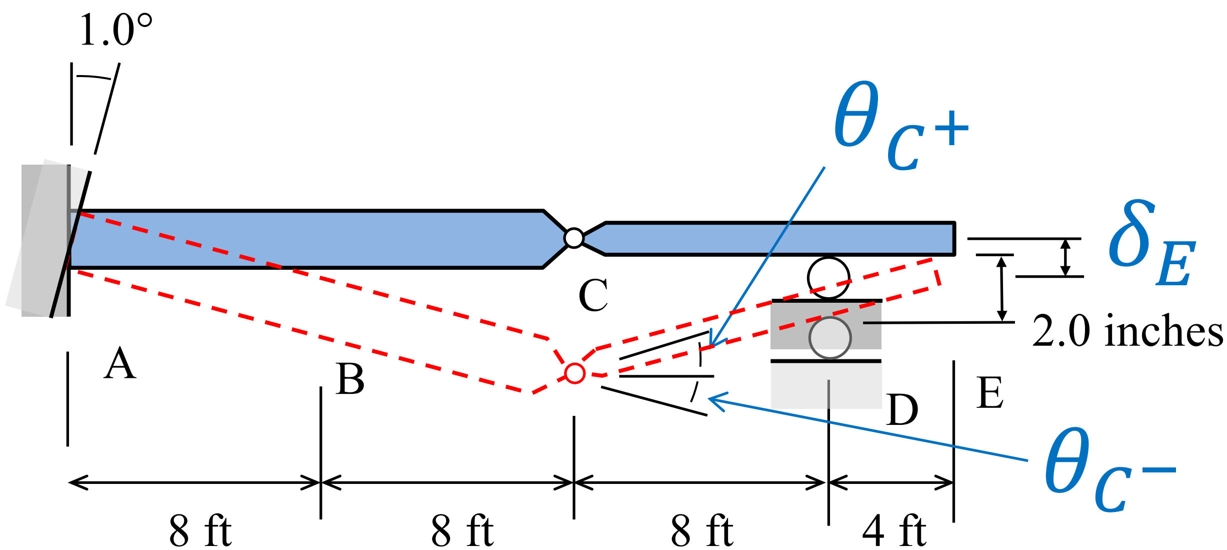
Method of Virtual Work
Beam Deflection Example
Support Movement

Steven Vukazich
San Jose State University

Beam Support Movement Deflection Example



The overhanging beam, from our previous example, has a fixed support at A, a roller support at C and an internal hinge at B.

EI_{ABC} = 2,000,000 k-in^2 and EI_{CDE} = 800,000 k-in^2

For the support movements shown, find the following:

1. The vertical deflection at point E;
2. The slope just to the left of the internal hinge at C;
3. The slope just to the right of the internal hinge at C

Recall the General Form of the Principle of Virtual Work

Real Deformation

$$Q\,\delta_P + \sum R_Q\,\delta_s = \iint F_Q\,dL_P$$

Virtual Loads

External Work due to Support Settlements

General Form for Bending Deformation

$$Q\,\delta_P + \sum R_Q\,\delta_s = \int_0^L M_Q \frac{M_P}{EI}\,dx$$

Principle of Virtual Work for Bending Deformation

Real deformation of interest

Real support movements

$$Q\,\delta_P + \sum R_Q\,\delta_s = \int_0^L M_Q \frac{M_P}{EI}\,dx$$

Virtual load

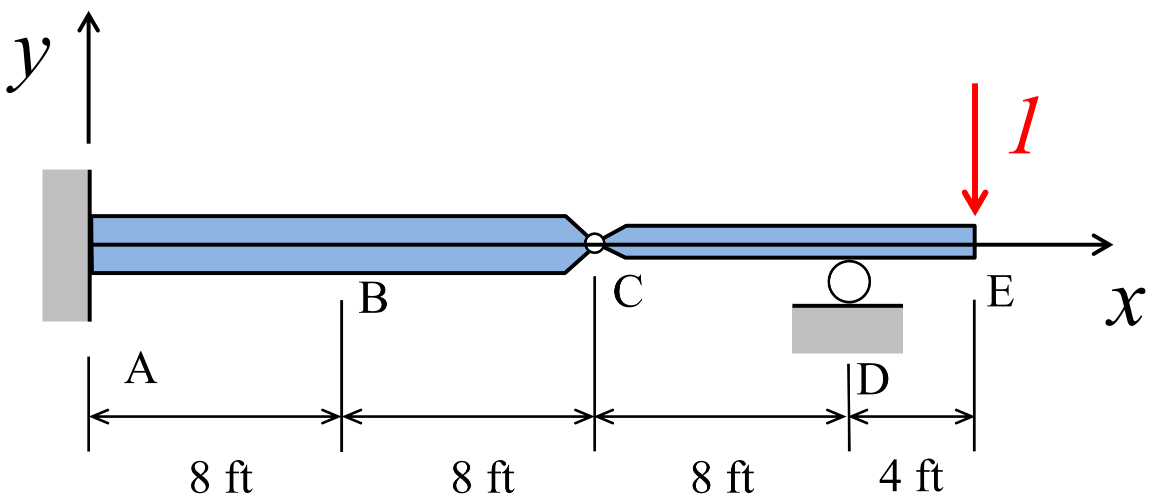
Virtual support reactions

Internal work = 0 for this problem

For this problem, there is only support movement causing deformation, so the internal work term is zero.

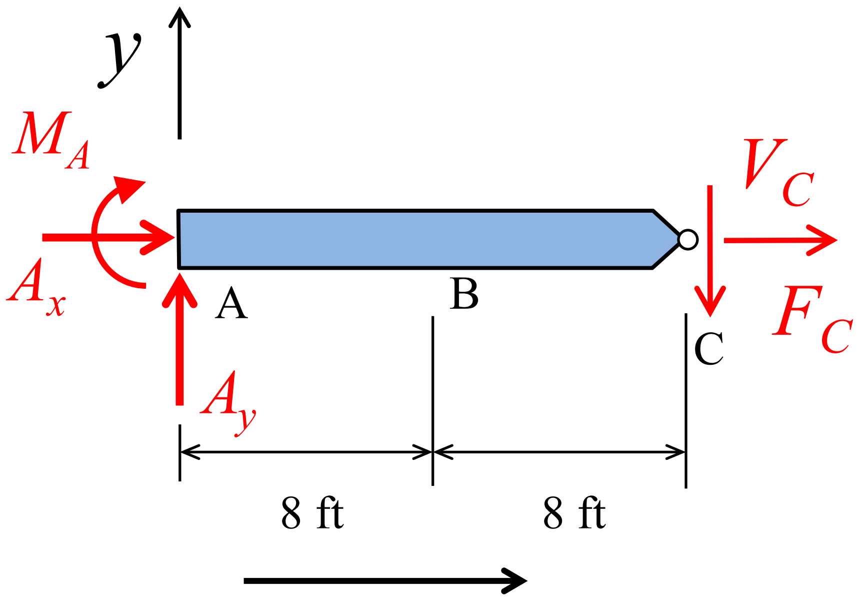
In order to find the external work due to support movement, we need to find the support reaction for the virtual system.

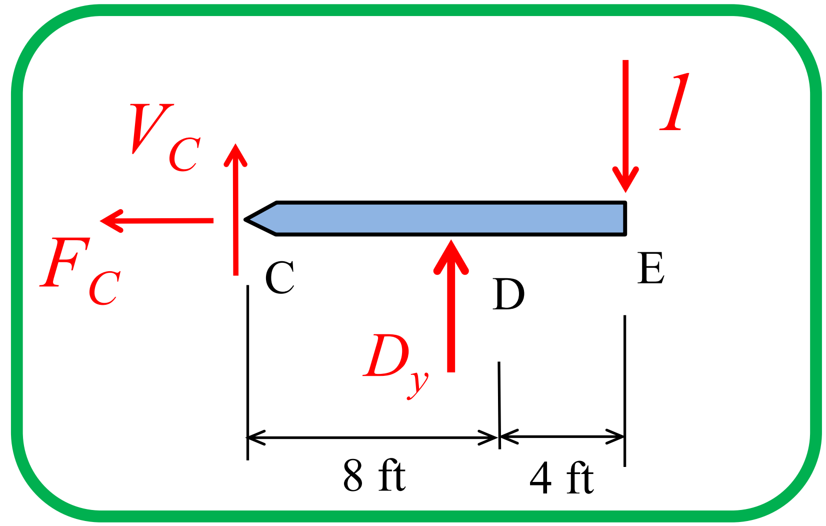
Virtual System to Measure the Deflection at Point E



From an equilibrium analysis, find the support reactions for the virtual system: R_Q

Find the Support Reactions for the Virtual System





$\circlearrowleft + \sum M_A = 0 \rightarrow$ $M_A = 8$ ft

$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

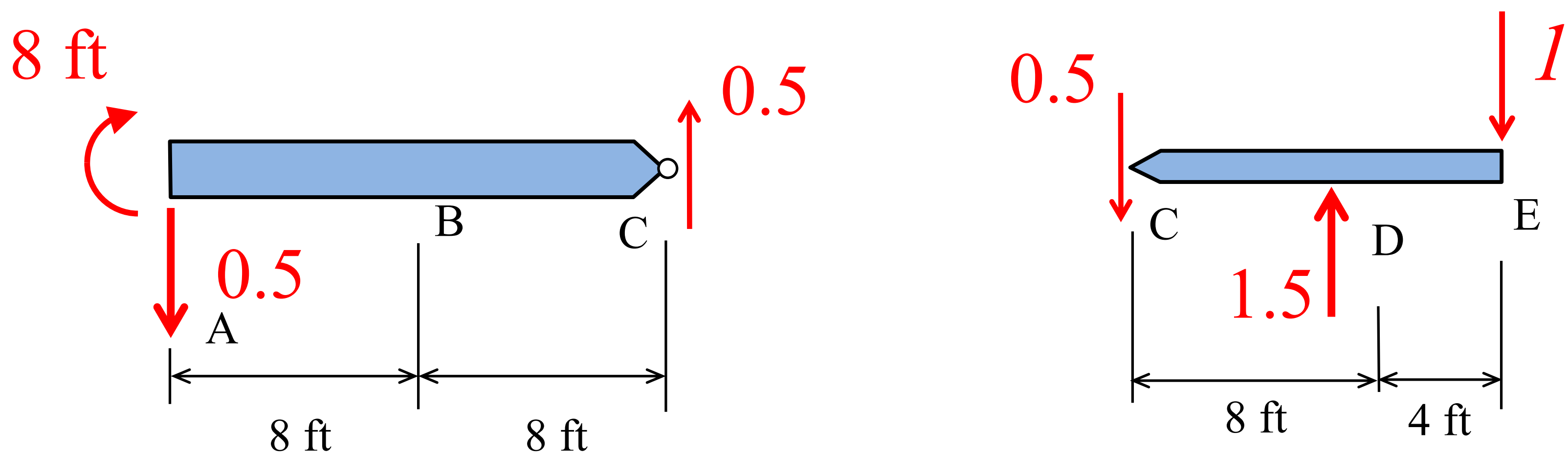
$+\uparrow \sum F_y = 0 \rightarrow$ $A_y = -0.5$

$\circlearrowleft + \sum M_C = 0 \rightarrow$ $D_y = 1.5$

$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$

$+\uparrow \sum F_y = 0 \rightarrow$ $V_C = -0.5$

Support Reactions for the Virtual System



Evaluate the Virtual Work Expression

$$1 \cdot \delta_E + \sum R_Q \, \delta_s = 0$$

$$1 \cdot \delta_E + M_{QA}\theta_{QA} + R_{QD}\delta_{QD} = 0$$

Need to convert θ_{QA} to radians

$$\theta_{QA} = 1° \left(\frac{\pi \text{ radians}}{180°}\right) = 0.017453 \text{ radians}$$

$$\delta_E + (8 \text{ ft})(0.017453)\left(\frac{12 \text{ in}}{\text{ft}}\right) - (1.5)(2.0 \text{ in}) = 0$$

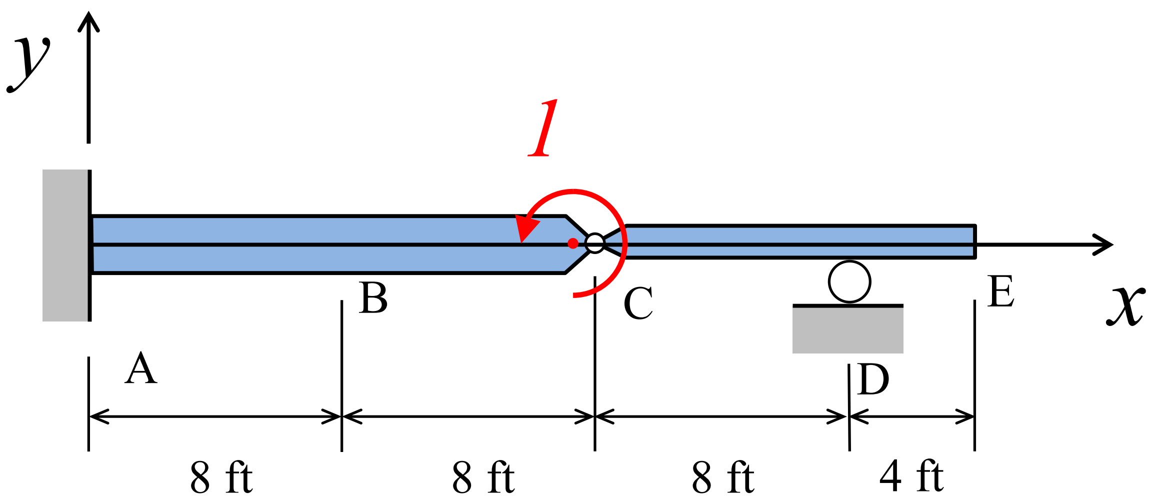
$$\delta_E + 1.6755 \text{ in} - 3.0 \text{ in} = 0$$

$$\delta_E = 1.325 \text{ in}$$

Positive result, so deflection is in the same direction as the virtual unit load

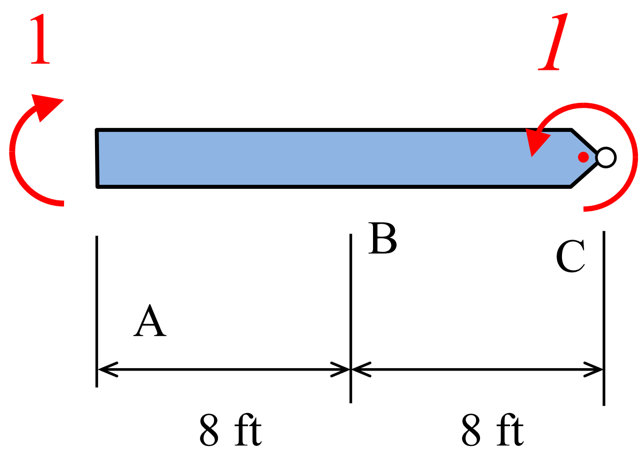
$$\delta_E = 1.325 \text{ in downward}$$

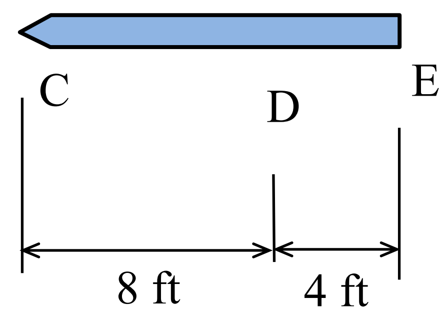
Virtual System to measure the Rotation Just to the Left of Point C



From an equilibrium analysis, find the support reactions for the virtual system: R_Q

Support Reactions for the Virtual System





Evaluate the Virtual Work Expression

$$1 \cdot \theta_{C^-} + \sum R_Q \, \delta_s = 0$$

$$1 \cdot \theta_{C^-} + M_{QA}\theta_{QA} + R_{QD}\delta_{QD} = 0$$

Need to convert θ_{QA} to radians

$$\theta_{QA} = 1° \left(\frac{\pi \text{ radians}}{180°}\right) = 0.017453 \text{ radians}$$

$$\theta_{C^-} + (1)(0.017453) = 0$$

$$\theta_{C^-} = -0.017453 \text{ rad} = -1°$$

Negative result, so deflection is in the opposite direction as the virtual unit moment

$$\theta_{C^-} = 0.01743 \text{ radians} = -1° \text{ clockwise}$$

Evaluate Product Integrals

$$\int_0^{L_{ABC}} M_Q\, M_P dx = \left(-338 + 114\ \text{k–ft}^2\right)\left(\frac{12^2\ \text{in}^2}{\text{ft}^2}\right) = -32{,}256\ \text{k–in}^2$$

$$\int_0^{L_{CDE}} M_Q\, M_P dx = 0$$

$$1 \cdot \theta_{C^-} = \frac{1}{EI_{ABC}} \int_0^{L_{ABC}} M_Q\, M_P dx + \frac{1}{EI_{CDE}} \int_0^{L_{CDE}} M_Q\, M_P dx$$

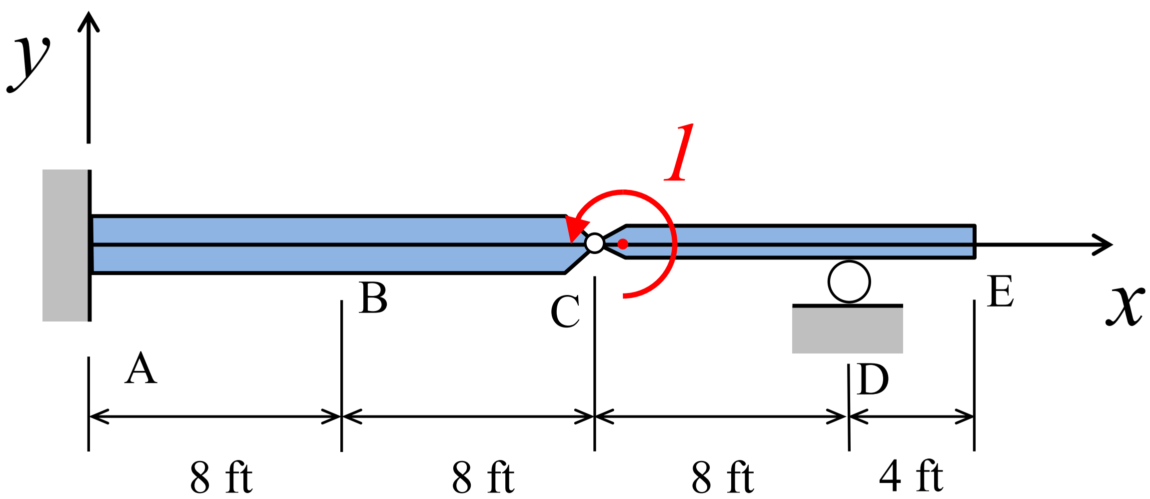
$$\theta_{C^-} = \frac{-32{,}256\ \text{k–in}^2}{2{,}000{,}000\ \text{k–in}^2} + \frac{0}{800{,}000\ \text{k–in}^2}$$

$$\theta_{C^-} = -0.0161\ \text{rad} + 0 = -\ 0.0161\ \text{rad}$$

Negative result, so rotation is in the opposite direction of the virtual unit moment

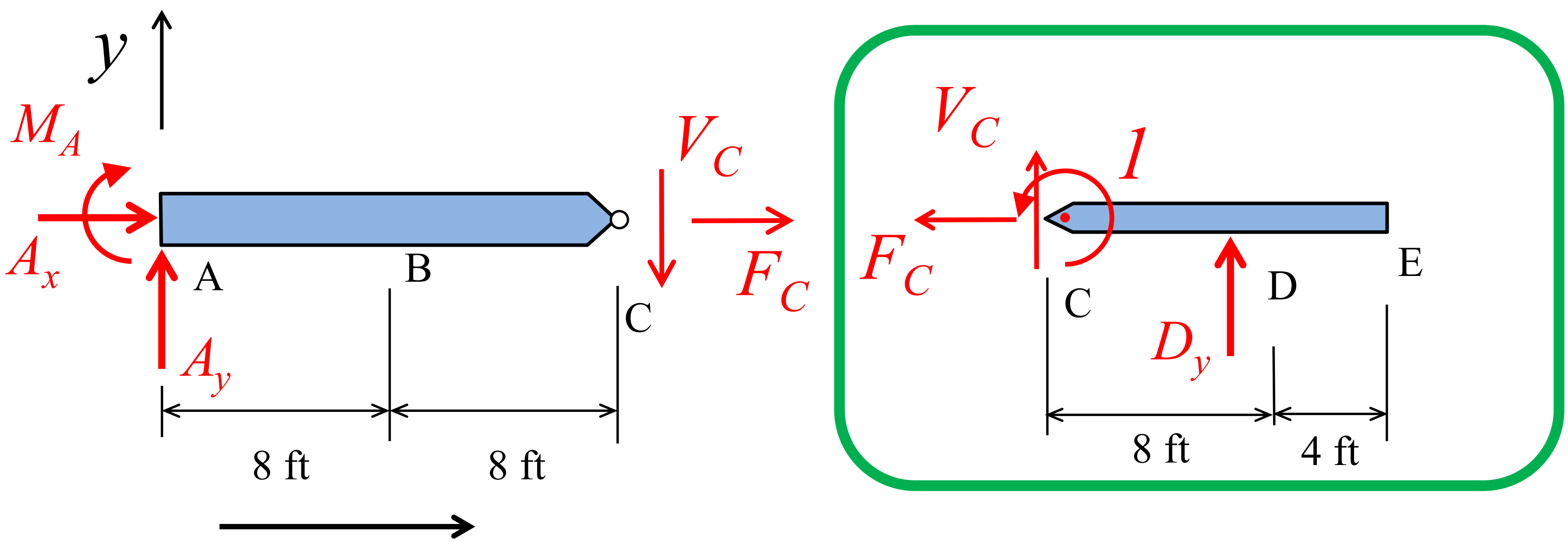
$$\theta_{C^-} = 0.0161\ \text{radians clockwise}$$

Virtual System to Measure the Rotation Just to the Right of Point C



From an equilibrium analysis, find the support reactions for the virtual system: R_Q

Find the Moment Diagram for the Virtual System



$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = -2$

$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

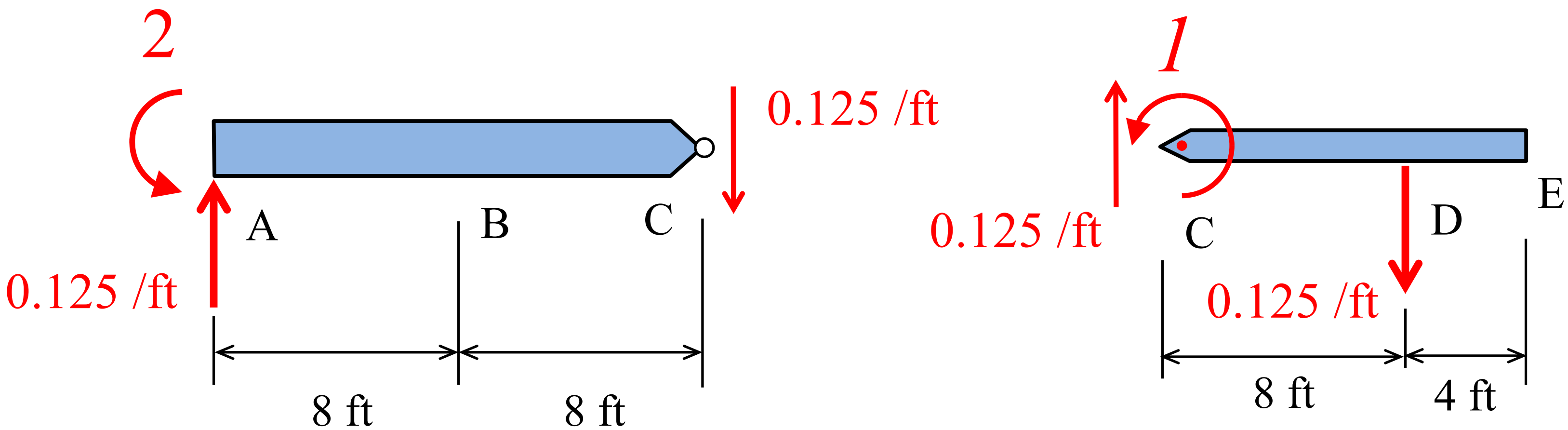
$+\uparrow \sum F_y = 0 \rightarrow A_y = 0.125\ /\text{ft}$

$\circlearrowleft + \sum M_C = 0 \rightarrow D_y = -0.125\ /\text{ft}$

$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$

$+\uparrow \sum F_y = 0 \rightarrow V_C = 0.125\ /\text{ft}$

Support Reactions for the Virtual System



Evaluate the Virtual Work Expression

$$1 \cdot \theta_{C^+} + \sum R_Q \, \delta_s = 0$$

$$1 \cdot \theta_{C^+} + M_{QA}\theta_{QA} + R_{QD}\delta_{QD} = 0$$

Need to convert θ_{QA} to radians

$$\theta_{QA} = 1^\circ \left(\frac{\pi \text{ radians}}{180^\circ}\right) = 0.017453 \text{ radians}$$

$$\theta_{C^+} - (2)(0.017453) + (0.125\ /\text{ft})(2.0 \text{ in})\left(\frac{\text{ft}}{12 \text{ in}}\right) = 0$$

$$\theta_{C^+} - 0.034906 \text{ rad} + 0.020833 \text{ rad} = 0$$

$$\theta_{C^+} = 0.01407 \text{ rad}$$

Negative result, so deflection is in the opposite direction as the virtual unit moment

$$\theta_{C^+} = 0.01407 \text{ radians counter−clockwise}$$

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